

AD-A065 785

FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO  
THE OPTIMUM COMPENSATION OF TEMPERATURE INSTABILITY CAUSED BY M--ETC(U)  
JUL 78 W JIA, P ZHANG  
FTD-ID(RS)T-0920-78

F/6 9/5

UNCLASSIFIED

NL

| OF |  
AD  
A085785



1

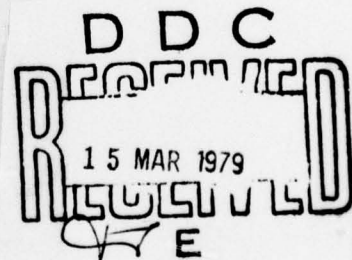
## FOREIGN TECHNOLOGY DIVISION



THE OPTIMUM COMPENSATION OF TEMPERATURE INSTABILITY CAUSED BY  
MAGNETOCRYSTALLINE ANISOTROPY FIELD IN YIG MICROWAVE DEVICES

By

Jia Wei-yi and Zhang Peng-xiang



Approved for public release;  
distribution unlimited.

78 12 22 305

AD-A065785

# EDITED TRANSLATION

FTD-ID(RS)T-0920-78

5 July 1978

MICROFICHE NR: *FD-78C-000917*

THE OPTIMUM COMPENSATION OF TEMPERATURE INSTABILITY  
CAUSED BY MAGNETOCRYSTALLINE ANISOTROPY FIELD  
IN YIG MICROWAVE DEVICES

By: Jia Wei-yi and Zhang Peng-xiang

English pages: 22

Source: Acta Physica Sinica, Vol 25, no. 3,  
May 1976, pp. 254-264

Country of Origin : China

Translated by: LINGUISTIC SYSTEMS, INC.  
F33657-76-D-0389  
H.P. Lee

Requester: FTD/TQH

Approved for public release;  
distribution unlimited.

DTIS	Write Section <input checked="" type="checkbox"/>
DDC	B.11 Section <input type="checkbox"/>
DIS	<input type="checkbox"/>
DISTRIBUTION/AVAILABILITY CODES	
SPECIAL	
<i>A</i>	

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP-AFB, OHIO.

The Optimum Compensation of Temperature Instability Caused by  
Magnetocrystalline Anisotropy Field in YIG Microwave Devices

Jia Wei-yi and Zhang Peng-xiang  
(Institute of Physics, Academia Sinica)

Abstract

More precise analytic expressions for ferromagnetic resonance of single crystals are calculated, the approximation being carried to terms quadratic in the magnetocrystalline anisotropy field. On the basis of these, the problem of the compensation of the temperature instabilities of single crystal YIG microwave devices, caused by the variations of the anisotropy field, is discussed. Significant corrections to the temperature compensation directions that have been in use are introduced. Formulae for the calculation of the temperature characteristics of YIG devices with the applied magnetic field in the corrected compensation directions are given.

In single crystal YIG microwave devices which are based upon the theory of ferromagnetic resonance, such as solid signal sources, wave filter, YIG sphere shows great effect of  $Q$  harmonic oscillator. Besides additional magnetic field control, the harmonic oscillation frequency of YIG sphere depends upon magnetocrystalline anisotropy field ( $K_1/M$ ). Because the latter is the temperature function, it causes temperature instability in the devices. There have been many articles which point out that if the linear term of anisotropy field, which have great effect on some crystallographic axis directions, is zero, and if these directions are the operation directions of crystals, the temperature stability will be greatly improved. These crystallographic axes are styled as temperature compensation axes (1-3). Even if these crystallographic axes are operating



directions, there still are quadratic terms of anisotropy field and their impacts are considerable, so there is great temperature instability left over.

Tokheim and Johnson have discussed and analyzed this problem in detail<sup>(4)</sup>. According to the results of their analysis and experiments and the factors which affect temperature stability as they have listed, the major one is anisotropy field. Even if the YIG sphere uses temperature compensation axes as operation axes, the situation remains so, and even more so in microwave frequency. The results of their experiments further indicate that the left-over temperature instability which has not been completely compensated cannot be explained by the first approximation theory of ferromagnetic resonance of the past. For instance, in their experiments, they discovered that the temperature compensation axial direction  $\langle 8013 \rangle$  in crystal plane  $\{010\}$  is much better than direction  $\langle 225 \rangle$  in plane  $\{110\}$ , and it is just contrary to the prediction of first approximation theory that direction  $\langle 225 \rangle$  is the best. The drifting rate of this direction prediction is one quantity level lower than the experiment value. In addition, to these two different kinds of directions, the temperature coefficient predicted by the theory is positive, that means when temperature rises, the harmonic oscillation frequency rises, but the temperature coefficient indicated in their experiment is negative. Those writers have pointed out that the source of this contradiction lies in the fact that theories in the past did not seriously consider the unparalleled situation of the saturated magnetization of the sample and the additional magnetic field. But the

ferromagnetic resonance formula suggested by those writers is rather complicated. It contains 30 obscure angle functions, so it is not a formula which can be easily discussed and solved. Moreover they mistake the term  $r/f_1/f_2$  in expression (here  $r$  is the same as  $\epsilon$ ,  $f_1 = \cos \theta$ ,  $f_2 = \sin \theta$  we use in this article) as the main factor of contradiction that has never been touched by the past theories. In fact, however, the major part of this term is first approximation term, and it has been discussed in the first approximation theory by Bickford<sup>(6)</sup> and Artman<sup>(7)</sup>.

For having easy discussions on problems, in this article, we suggest secondary small-angle approximation, from which a more precise analytic formula of ferromagnetic resonance can be deduced, and the contradiction between theory and practice may be thereby solved. It can also prove that among all temperature compensation directions, direction  $\langle 8013 \rangle$  in crystal plane  $\{010\}$  is the best compensation direction, and direction  $\langle 225 \rangle$  in crystal plane  $\{1\bar{1}0\}$  is a very poor one. The temperature compensation effect of the former is two times better than the latter. The theoretical results also indicate that, under low frequency, the impact of anisotropy filed will increase relatively and that there should be a great correction to the temperature compensation direction and the shaft angle  $\langle 001 \rangle$ . Based on these results, the temperature characteristics of YIG devices, when the additional magnetic field is in compensation direction, can be easily calculated. These results may be of great significance to the design and adjustment of YIG devices. And the ferromagnetic resonance formula deduced in this article will also be significant to the precise test of magnetism.

# 1. The General Expression of Ferromagnetic Resonance Under Secondary Small-Angle Approximation

In the earlier single crystal ferromagnetic resonance theories, which dealt with the impact of magnetocrystalline anisotropy, have supposed the approximation of saturated magnetization  $M$  paralleling with additional constant magnetic field  $H^{(5)}$ . Later they came to recognize that  $M$  in fact is not parallel with  $H$ , and under small-angle approximation, the included angle between  $M$  and  $H$  is thought to be in direct ratio to  $|K_1|/H$ . The earlier resonance expression of crystal plane  $\{100\}$  and  $\{110\}$  has been corrected and the result of the correction is the one that is used widely today. But under precise test and in practical application, the degree of approximation of those results has more and more proved unable to satisfy requirement. Under general condition,  $M$  is considered not parallel with  $H$ , and it is rather difficult to have a precise expression of ferromagnetic resonance. In this article, we use secondary small-angle approximation and take the impact of term  $(K_1/H)^2$  into account when we calculate the included angle of  $M$  and  $H$ . But we still suppose that the additional constant magnetic field is much larger than anisotropy field, and that the included angle of saturated magnetization  $M$  (spherical coordinate is  $\theta, \varphi$ ) and additional magnetic field (spherical coordinate is  $\theta_0, \varphi_0$ ) is small. Most of the practical situations can satisfy this condition. The basic equation of ferromagnetic resonance is <sup>(8)</sup>

$$\left(\frac{\omega}{\gamma}\right)^2 = \frac{1}{M^2 \sin^2 \theta_0} \{F_{\theta\theta} F_{\varphi\varphi} - F_{\theta\varphi}^2\}_{\theta_0, \varphi_0}, \quad (1)$$

Here  $\omega$  is harmonic oscillation frequency,  $\gamma = \frac{\gamma'}{2\pi} = 2.80 \times 10^4 \text{ Oe}^{-1} \text{ sec}^{-1}$   
 $\gamma'$  is rotary magnet ratio;  $\theta_0, \varphi_0$  indicates equilibrium position of  $M$  and



$F_{00}$  is second order differential quotient of free energy. Free energy is

$$\begin{aligned} F &= E - M \cdot H \\ &= E - MH \{ \sin \theta_0 \sin \theta \cos(\varphi - \varphi_0) \\ &\quad + \cos \theta_0 \cos \theta \}, \end{aligned} \quad (2)$$

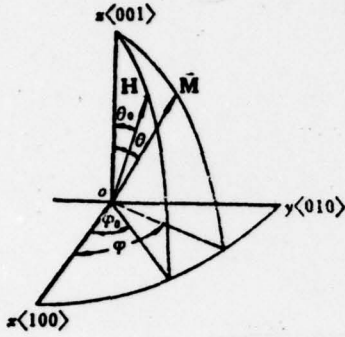


Figure 1 Coordinate of saturated magnetization  $M$  and additional magnetic field  $H$

The coordinate relation as illustrated in Figure 1.  $E$  indicates anisotropy energy. It can be magnetocrystalline anisotropy energy and can also include anisotropy energy of other kind, such as shape anisotropy energy. Let

$$\begin{aligned} \theta - \theta_0 &= \epsilon, \\ \varphi - \varphi_0 &= \eta. \end{aligned}$$

that  
From the first order differential quotient of free energy is zero,  $M$  equilibrium position will have

$$\frac{\partial F}{\partial \theta} = \frac{\partial E}{\partial \theta} - MH \{ \sin \theta_0 \cos \theta \cos(\varphi - \varphi_0) - \cos \theta_0 \sin \theta \} = 0, \quad (3)$$

$$\frac{\partial F}{\partial \varphi} = \frac{\partial E}{\partial \varphi} + MH \sin \theta_0 \sin \theta \sin(\varphi - \varphi_0) = 0. \quad (4)$$

In order to have equilibrium position, namely  $\epsilon$  &  $\eta$ , it must make stage number expansion of equation (3) and (4) at  $\theta_0$  &  $\varphi_0$  and maintain the quadratic term of  $\epsilon$  and  $\eta$ :

$$\begin{aligned} -\epsilon - \frac{1}{2} \sin \theta_0 \cos \theta_0 \eta^2 &= \frac{1}{MH} \left\{ \frac{\partial E}{\partial \theta} + \frac{\partial^2 E}{\partial \theta^2} \epsilon + \frac{1}{2} \frac{\partial^3 E}{\partial \theta^3} \epsilon^2 \right. \\ &\quad \left. + \frac{\partial^2 E}{\partial \theta \partial \varphi} \eta + \frac{1}{2} \frac{\partial^3 E}{\partial \theta \partial \varphi^2} \eta^2 + \frac{\partial^3 E}{\partial \theta^2 \partial \varphi} \epsilon \eta \right\}_{\theta_0, \varphi_0}, \end{aligned} \quad (5)$$

$$\begin{aligned} -\sin^2 \theta_0 \eta - \cos \theta_0 \sin \theta_0 \epsilon \eta &= \frac{1}{MH} \left\{ \frac{\partial E}{\partial \varphi} + \frac{\partial^2 E}{\partial \theta \partial \varphi} \epsilon \right. \\ &\quad \left. + \frac{1}{2} \frac{\partial^3 E}{\partial \theta^2 \partial \varphi} \epsilon^2 + \frac{\partial^2 E}{\partial \varphi^2} \eta + \frac{1}{2} \frac{\partial^3 E}{\partial \varphi^3} \eta^2 + \frac{\partial^3 E}{\partial \theta \partial \varphi^2} \epsilon \eta \right\}_{\theta_0, \varphi_0}. \end{aligned} \quad (6)$$



Under first approximation, from equation (5) and (6) it can have

$$\begin{aligned}\epsilon &= -\frac{1}{MH} \frac{\partial E}{\partial \theta}, \\ \eta &= -\frac{1}{MH \sin^2 \theta_0} \frac{\partial E}{\partial \varphi}.\end{aligned}\quad (7)$$

Now  $\epsilon$  and  $\eta$  are in direct ratio to the linear terms of anisotropy field. Under the requirement for maintaining the square terms of anisotropy field and ignoring the higher ones, the quadratic terms of  $\epsilon$  and  $\eta$  on the right hand side of equation (5) and (6) can be left off because they are in fact the cubic terms of anisotropy field. To seek solution of equation (5) and (6) uses substitution method. To substitute equation (7) for the quadratic terms of  $\epsilon$  and  $\eta$  in equation (5) and (6) and then to seek solution of the new equation according to the degree of approximation, it will have

$$\epsilon = -\frac{1}{MH} \frac{\partial E}{\partial \theta} + \frac{1}{M^2 H^2} \left\{ \frac{\partial E}{\partial \theta} \frac{\partial^2 E}{\partial \theta^2} - \frac{\cos \theta_0}{2 \sin^3 \theta_0} \left( \frac{\partial E}{\partial \varphi} \right)^2 + \frac{1}{\sin^2 \theta_0} \frac{\partial E}{\partial \varphi} \frac{\partial^2 E}{\partial \theta \partial \varphi} \right\}_{\theta_0, \varphi_0}, \quad (8)$$

$$\begin{aligned}\eta &= -\frac{1}{MH \sin^2 \theta_0} \frac{\partial E}{\partial \varphi} - \frac{1}{M^2 H^2} \left\{ \frac{\cos \theta_0}{\sin^3 \theta_0} \frac{\partial E}{\partial \theta} \frac{\partial E}{\partial \varphi} - \frac{1}{\sin^2 \theta_0} \frac{\partial E}{\partial \theta} \frac{\partial^2 E}{\partial \theta \partial \varphi} \right. \\ &\quad \left. - \frac{1}{\sin^4 \theta_0} \frac{\partial E}{\partial \varphi} \frac{\partial^2 E}{\partial \varphi^2} \right\}_{\theta_0, \varphi_0}.\end{aligned}\quad (9)$$

If taking equation (8) and (9) continuously to substitute for the quadratic terms of  $\epsilon$  and  $\eta$  in equation (5) and (6), the result will show that under the secured degree of approximation, there will add no more new terms. Equation (8) and equation (9) are already full expressions.

A further calculation will be making stage number expansion of the free energy differential quotient of equation (1) at the vicinity of  $\theta_0$  &  $\varphi_0$ .

$$\begin{aligned}
\frac{\partial^2 F}{\partial \theta^2} &= \left\{ \frac{\partial^2 E}{\partial \theta^2} + \frac{\partial^2 E}{\partial \theta^2} \epsilon + \frac{\partial^3 E}{\partial \theta^2 \partial \varphi} \eta + MH \left( 1 - \frac{1}{2} \epsilon^2 - \frac{1}{2} \sin^2 \theta_0 \eta^2 \right) \right\}_{\theta_0, \varphi_0}, \\
\frac{\partial^2 F}{\partial \varphi^2} &= \left\{ \frac{\partial^2 E}{\partial \varphi^2} + \frac{\partial^3 E}{\partial \varphi^3} \eta + \frac{\partial^3 E}{\partial \theta \partial \varphi^2} \epsilon + MH \sin^2 \theta_0 \left( 1 + \cot \theta_0 \epsilon - \frac{1}{2} \epsilon^2 - \frac{1}{2} \eta^2 \right) \right\}_{\theta_0, \varphi_0}, \\
\frac{\partial^2 F}{\partial \theta \partial \varphi} &= \left\{ \frac{\partial^2 E}{\partial \theta \partial \varphi} + \frac{\partial^3 E}{\partial \theta \partial \varphi^2} \eta + \frac{\partial^3 E}{\partial \theta^2 \partial \varphi} \epsilon + MH (\sin \theta_0 \cos \theta_0 \eta - \sin^2 \theta_0 \epsilon \eta) \right\}_{\theta_0, \varphi_0}, \\
\frac{1}{\sin^2 \theta_0} &= \frac{1}{\sin^2 \theta_0} \left( 1 - \frac{2 \cos \theta_0}{\sin \theta_0} \epsilon + \frac{1 + 2 \cos^2 \theta_0}{\sin^2 \theta_0} \epsilon^2 \right).
\end{aligned} \tag{10}$$

To place these relations in equation (1) and hold square terms of anisotropy field; and to leave off higher terms by using equation (8) and (9), we finally have

$$\begin{aligned}
\left( \frac{\omega}{\gamma} \right)^2 &= H^2 + \frac{H}{M} \left\{ \frac{\partial^2 E}{\partial \theta^2} + \frac{1}{\sin^2 \theta} \frac{\partial^2 E}{\partial \varphi^2} + \cot \theta \frac{\partial E}{\partial \theta} \right\} \\
&+ \frac{1}{M^2} \left\{ \cot^2 \theta \left( \frac{\partial E}{\partial \theta} \right)^2 - \frac{1}{\sin^2 \theta} \frac{\partial E}{\partial \theta} \frac{\partial^2 E}{\partial \theta \partial \varphi^2} + \frac{2 \cos \theta}{\sin^3 \theta} \frac{\partial E}{\partial \theta} \frac{\partial^2 E}{\partial \varphi^2} - \frac{\partial E}{\partial \theta} \frac{\partial^3 E}{\partial \theta^3} \right. \\
&- \frac{1}{\sin^4 \theta} \left( \frac{\partial E}{\partial \varphi} \right)^2 + \frac{\cos \theta}{\sin^3 \theta} \frac{\partial E}{\partial \varphi} \frac{\partial^2 E}{\partial \theta \partial \varphi} - \frac{1}{\sin^4 \theta} \frac{\partial E}{\partial \varphi} \frac{\partial^3 E}{\partial \varphi^3} - \frac{1}{\sin^2 \theta} \frac{\partial E}{\partial \varphi} \frac{\partial^3 E}{\partial \theta^2 \partial \varphi} \\
&\left. + \frac{1}{\sin^2 \theta} \frac{\partial^2 E}{\partial \theta^2} \frac{\partial^2 E}{\partial \varphi^2} - \frac{1}{\sin^2 \theta} \left( \frac{\partial^2 E}{\partial \theta \partial \varphi} \right)^2 \right\}.
\end{aligned} \tag{11}$$

Notice, we have excluded the angular coordinate "0" of  $\theta$  &  $\varphi$ , here as well as in the following, they indicate additional magnetic fields. Equation (11) is the general equation of ferromagnetic resonance in any crystallographic axial direction under secondary small-angle approximation, and precisely it represents square terms of anisotropy. In it, the part of linear terms of anisotropy field and the results of first approximation are the same<sup>(7)</sup>, but to the part of square terms many new components are added. This indicates that the secondary approximation gives some correction to the square terms. And in the following, this kind of correction will prove very important.

## 2. The Resonance Expression of Single Crystal Ferromagnet of Cubic Structure

For a single crystal sphere of ferromagnet, there is no relationship between its demagnetization and direction, so  $E$  in equation (2) only contains the property of magnetocrystalline anisotropy. For a ferromagnet of cubic symmetry:

$$E = \frac{K_1}{4} (\sin^2 2\theta + \sin^4 \theta \sin^2 2\varphi) + \frac{K_2}{4} \sin^4 \theta \cos^2 \theta \sin^2 2\varphi. \quad (12)$$

Placing its corresponding differential quotient of each order into equation (11), we can have ferromagnetic resonance expression of the spherical sample of cubic symmetry ferromagnet at any crystallographic axial direction:

$$\left(\frac{\omega}{\gamma}\right)^2 = H^2 + H \left( \frac{K_1}{M} f_1 + \frac{K_2}{M} f_3 \right) + \frac{K_1^2}{M^2} f_2 + \frac{K_1 K_2}{M^2} f_4 + \frac{K_2^2}{M^2} f_5. \quad (13)$$

Here  $f_1$  is the function of additional magnetic field direction  $(\theta, \varphi)$ . For the YIG ferrite materials used for single crystal devices, generally there is  $\left| \frac{K_1}{M} \right| \gg \left| \frac{K_2}{M} \right|$ . For simplicity, in the following, only three functions are given:

$$f_1 = \frac{1}{16} [9 + 20 \cos 2\theta + 35 \cos 4\theta + (15 - 20 \cos 2\theta + 5 \cos 4\theta) \cos 4\varphi], \quad (14)$$

$$\begin{aligned} f_2 = & \left( \frac{3385}{1024} + \frac{63}{64} \cos 2\theta + \frac{405}{256} \cos 4\theta - \frac{27}{64} \cos 2\theta \cos 4\theta - \frac{1485}{1024} \cos 8\theta \right) \\ & + \left( \frac{21}{256} - \frac{27}{16} \cos 2\theta + \frac{117}{64} \cos 4\theta + \frac{3}{16} \cos 2\theta \cos 4\theta - \frac{105}{256} \cos 8\theta \right) \cos 4\varphi \\ & + \left( \frac{-525}{1024} + \frac{45}{64} \cos 2\theta - \frac{105}{256} \cos 4\theta + \frac{15}{64} \cos 2\theta \cos 4\theta - \frac{15}{1024} \cos 8\theta \right) \cos 8\varphi, \end{aligned} \quad (15)$$

$$\begin{aligned} f_3 = & \frac{1}{64} (1 + 13 \cos 2\theta + 7 \cos 4\theta - 21 \cos 2\theta \cos 4\theta) \\ & + \frac{1}{64} (15 - 13 \cos 2\theta - 23 \cos 4\theta + 21 \cos 2\theta \cos 4\theta) \cos 4\varphi. \end{aligned} \quad (16)$$

When  $\varphi = 0$ , equation (13) will be transformed into ferromagnetic formula

in crystal plane  $\{010\}$  :

$$\begin{aligned} \left(\frac{\omega}{\gamma}\right)^2 = & H^2 + H \left[ (3 + 5 \cos 4\theta) \frac{K_1}{2M} + (1 - \cos 4\theta) \frac{K_2}{4M} \right] \\ & + [23 + 24 \cos 4\theta - 15 \cos 8\theta] \frac{K_1^2}{8M^2}; \end{aligned} \quad (17)$$

When  $\varphi = \pi/4$ , equation will be simplified into ferromagnetic resonance formula in crystal plane  $\{110\}$  :

$$\begin{aligned} \left(\frac{\omega}{\gamma}\right)^2 = & H^2 + H \left[ (-3 + 20 \cos 2\theta + 15 \cos 4\theta) \frac{K_1}{8M} \right. \\ & \left. + (-7 + 13 \cos 2\theta + 15 \cos 4\theta - 21 \cos 2\theta \cos 4\theta) \frac{K_2}{32M} \right] \\ & + [347 + 432 \cos 2\theta - 84 \cos 4\theta - 48 \cos 2\theta \cos 4\theta - 135 \cos 8\theta] \frac{K_1^2}{128M^2}. \end{aligned} \quad (18)$$

Here the part of function  $f_4$  and  $f_5$  are omitted. It shows clearly once again that the first terms of anisotropy field and the first approximation theory in the past are identical<sup>(7)</sup>, but the part of quadratic terms is very much different. What is worth pointing out is that the first terms and quadratic terms of anisotropy field which are contained in resonance formula of the main crystallographic axial directions  $\langle 100 \rangle$ ,  $\langle 110 \rangle$ ,  $\langle 111 \rangle$  obtained from calculation of equation (17) and (18), are identical with first approximation theory and the results calculated from the earlier hypothesis that  $M$  is parallel to  $H$ . This is something that can be predicted because in these main axial directions, the first order differential quotients of the extreme value of anisotropy energy are zero. From equation (8) and (9) it can be seen that  $\epsilon = \eta = 0$ , and this means that when additional magnetic field is along the main axial direction and as long as its strength is larger than saturated magnetic field, the magnetization vector is always parallel to it.



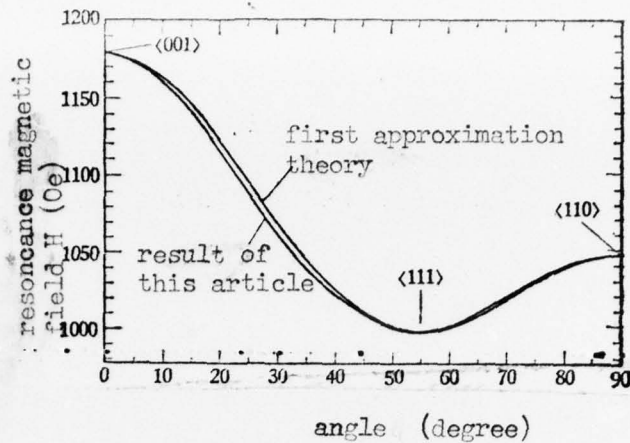


Figure 2 The relation of resonance magnetic field in crystal plane  $\{110\}$  and direction angle (a comparison of first approximation theory and the result of this article. In calculation, taking  $K_1/M = -55$  Oe (Ga-YIG).  $K_2/M$  is omitted. Frequency  $\omega = 3000$  MC.mc

Figure 2 illustrates a comparison of the relation of resonance magnetic field, according to the first approximation theory and the calculation in this article, and direction angle. The numerical value of anisotropy field is selected from room temperature value of YIG material which is mixed with Ga.  $K_2/M$  is omitted and frequency is 3000 mc. From Figure 2, it can be seen that when they are far away from main crystallographic axis direction, the first approximation theory and secondary approximation result are considerably different. The lower is the operating frequency, the larger is the anisotropy field, and the more striking is this difference.

### 3. The Elimination of the Impact of Magnetocrystalline Anisotropy Field in YIG Devices

As an application of the results discussed above, we here begin to discuss the problems of eliminating the temperature instability caused by magnetocrystalline anisotropy field.

## 1. The Track Plane of Temperature Compensation of Magnetocrystalline Anisotropy Field

From equation (13) it can be seen that the tuned frequency of YIG devices is contingent upon magnetocrystalline anisotropy field. The temperature change of the latter makes temperature instability of the devices, and the terms of  $K_1/M$  <sup>are</sup> the most effective factor. If the terms which are related to  $K_2/M$  are overlooked, equation (13) will change into

$$\left(\frac{\omega}{\gamma}\right)^2 = H^2 + H \frac{K_1}{M} f_1 + \frac{K_1^2}{M^2} f_2. \quad (19)$$

If  $H \gg \left|\frac{K_1}{M}\right|$ , then

$$\frac{\omega}{\gamma} = H + \frac{K_1}{M} \frac{f_1}{2} + \frac{K_1^2}{M^2 H} \left(\frac{f_1}{2} - \frac{f_2}{8}\right). \quad (20)$$

According to the definitions in many articles, the temperature compensation direction of anisotropy field is determined by that equation (14) is zero<sup>(9)</sup>:

$$9 + 20 \cos 2\theta + 35 \cos 4\theta + (15 - 20 \cos 2\theta + 5 \cos 4\theta) \cos 4\varphi = 0. \quad (21)$$

The track of compensation direction which is thus determined is usually a cone which takes  $\langle 001 \rangle$  as axis and has an opening angle of about  $30^\circ$ . From the cubic symmetry it can be known that the cone of this kind is of three pairs. Any of the crystal directions on these cones is temperature compensation direction, and its concrete spherical coordinate is determined by equation (21). For instance, if  $\varphi = 0$ , and the temperature compensation axis in crystal plane  $\{100\}$  is at  $\theta_c = 31^\circ 43'$ , it is almost direction  $\langle 8013 \rangle$ , and if  $\varphi = \pi/4$ , and the temperature compensation axis in crystal plane  $\{1\bar{1}0\}$  is at  $\theta_c = 29^\circ 40'$ , it is almost direction  $\langle 225 \rangle$ .

## 2. The Best Temperature Compensation Axis

From equation (13) it can be seen that in temperature compensation, there are anisotropy field terms dependent upon temperature. Among them, the term  $K_1^2/M^2$  is the most important one. Although it is much smaller than that which contains  $K_1/M$ , its impact can by no means be overlooked, especially under the condition of low temperature, because the changes of  $K_1^2/M^2H$  are very great, so it can seriously affect the stability of the devices. Therefore we must make a selection from among the temperature compensation directions and decide which is the best one. Since various experiments have proved that axis  $\langle 8013 \rangle$  is better than  $\langle 225 \rangle$ , there are some people who have raised question as whether there is another compensation direction which is better than what we know so far is the best. According to equation (19), such a question is to ask which is of the minimum  $f_2$  value among all the compensation axes. Under the restriction of equation (21) to seek extreme value from equation (15), the condition of extreme value obtained is

$$\sin 4\varphi = 0.$$

Within the range of  $\pi/2$ , the solution obtained is

$$\varphi = 0 \text{ or } \varphi = \pi/4.$$

It thus proves that  $\varphi = 0$  corresponding to  $f_2$  has minimum value and  $\varphi = \pi/4$  corresponding to  $f_2$  has maximum value. In order to satisfy equation (21), equation (15) must have a solution which is of positive value and not a zero. Thus it can be concluded theoretically that  $\langle 8013 \rangle$  in crystal plane  $\{010\}$  is the best temperature compensation axis, and direction  $\langle 225 \rangle$  in crystal plane  $\{\bar{1}10\}$  is the poorest temperature compensation axis. Substituting  $\varphi_1 = 31^\circ 43'$  and  $\varphi_2 = 29^\circ 40'$  in equation (17) and (18) respectively, we can have the ferromagnetic resonance formula of  $\langle 8013 \rangle$  and  $\langle 225 \rangle$ . When  $H \gg \left| \frac{K_1}{M} \right|$ ,



$$\langle 8013 \rangle \quad \frac{\omega}{\gamma} = H + 0.20 \frac{K_2}{M} + 0.80 \frac{K_1^2}{M^2 H}, \quad (22)$$

$$\langle 225 \rangle \quad \frac{\omega}{\gamma} = H - 0.04 \frac{K_2}{M} + 2.70 \frac{K_1^2}{M^2 H}. \quad (23)$$

In considering the impact of terms of  $K_1^2/M^2$ , now we can see that the temperature compensation effect of direction  $\langle 8013 \rangle$  is two times better than that of direction  $\langle 225 \rangle$ . To YIG devices, because  $K_2/M$  is negative, in direction  $\langle 8013 \rangle$  it can offset a part of term  $K_1^2/M^2$ , but the situation is just opposite in direction  $\langle 225 \rangle$ , so the temperature stability in direction  $\langle 8013 \rangle$  can elevate even higher. Because the coefficient of terms of  $K_1^2/M^2$  is positive, if the additional magnetic field has no change but its temperature rises, the tuned frequency will become lower following the reduction of  $\frac{K_1}{M}$ , and shows a negative temperature coefficient. The results mentioned above are all in accord with practical observations.

But to material, which has great  $\left| \frac{K_2}{M} \right|$  value, and the operating frequency, it makes  $\left| \frac{K_2}{M} \right| \gg \frac{K_1^2}{M^2 H}$ , the situation will be different. From equation (22) and (23), it can be seen that now direction  $\langle 225 \rangle$  is better than direction  $\langle 8013 \rangle$ . But the best temperature stability direction should be the solutions to equation (14) and (16) when they both are made equal to zero. The solution is close to  $\theta = 30^\circ$ ,  $\varphi = 32^\circ$ . When the additional magnetic field is along this direction, the impact of terms of  $K_1/M$  and  $K_2/M$  will vanish simultaneously. But to various YIG materials, of which  $\left| \frac{K_2}{M} \right|$  is not great, such situation never happens.

### 3. Results of First Approximation Calculation

When additional magnetic field is in the temperature compensation



direction in crystal plane of  $\{010\}$  kind or  $\{110\}$  kind, the ferromagnetic resonance frequency obtained from first approximation calculation is

$$\langle 8013 \rangle \quad \frac{\omega}{\gamma} = H + 0.20 \frac{K_1}{M} - 0.72 \frac{K_1^2}{M^2 H}, \quad (24)$$

$$\langle 225 \rangle \quad \frac{\omega}{\gamma} = H - 0.04 \frac{K_1}{M} - 0.108 \frac{K_1^2}{M^2 H}, \quad (25)$$

Thus we can conclude that direction  $\langle 225 \rangle$  is much better than direction  $\langle 8013 \rangle$ . Because the coefficient of terms of  $K_1^2/M^2$  is negative and  $\frac{d}{dT} \left( \frac{K_1^2}{M^2} \right) < 0$ , the temperature coefficient of frequency is positive. All these are not in accord with experiments.

#### 4. The Impact of Non-linear Terms

From equation (20), (22) and (23), it can be further seen that because of the impact of the terms of  $K_1^2/M^2$ , there is no longer a linear relation between additional magnetic field and resonance frequency. This fact brings damage to the degree of tuned linearity. Under low microwave frequency, the additional magnetic field becomes low, and the impact of non-linear terms of  $K_1^2/M^2 H$  is relatively increased. For instance, if the material used is  $\left| \frac{K_1}{M} \right| = 50$  Oe and the non-linear frequency is 1400mc, there will be a non-linear difference of 11.2mc at direction  $\langle 8013 \rangle$ , and 37.8mc at direction  $\langle 225 \rangle$ . It becomes clear now that operating at direction  $\langle 8013 \rangle$ , the degree of tuned linearity of YIG devices is much superior to that operation at direction  $\langle 225 \rangle$ .

#### 5. Further Correction of Temperature Compensation Direction

In most cases, the impact of  $K_1^2/M^2 H$  is great; especially to the

devices which have low microwave frequency. Because the additional magnetic field is low and the  $\frac{K_1}{M}$  value of the low saturated magnetization material used (such as Ga-YIG and BiCaVIG) at room temperature or lower than room temperature is greater than pure YIG, the result is that the impact of  $K_1^2/M^2H$  is very great and the temperature compensation of practical significance almost becomes non-existence. In order to diminish such impact, we should use  $K_1/M$  terms as compensation. The corrected temperature compensation direction can be decided by the following equations:

$$\frac{K_1}{M} f_1 + \frac{K_1^2}{M^2 H_p} f_1 = 0, \quad (26)$$

$$f_1 - m f_2 = 0, \quad (27)$$

$$m = \left| \frac{K_1}{M} \right| / H_p = \gamma \left| \frac{K_1}{M} \right| / \omega_p.$$

Here  $p$  is used to indicate the value which has been selected for determining compensation direction. In the practically used microwave single crystal devices,  $m = 0.01-0.2$ . If  $m > 0.2$ , the ferromagnetic resonance formula deduced in this article will become inapplicable. Then we have to consider the terms of  $(K_1/M)^3$ ; but when  $m$  reaches such great degree, temperature compensation direction of practical value no longer exists.

Figure 3 shows the relations of the corrected temperature compensation direction in crystal plane of  $\{010\}$  kind and  $\{\bar{1}\bar{1}0\}$  kind and direction coefficient  $m$ . It can be seen that when  $m = 0$ , compensation will occur at direction of  $\langle 8013 \rangle$  kind and  $\langle 225 \rangle$  kind. When  $m$  is amplified, the corrected temperature compensation direction and the included angle of  $\langle 001 \rangle$  axis become smaller, and the speed of following  $m$  change by the corrected compensation direction in crystal plane of  $\{010\}$  kind will be smaller than that in crystal plane of  $\{\bar{1}\bar{1}0\}$  kind. This is of course rather advantageous.

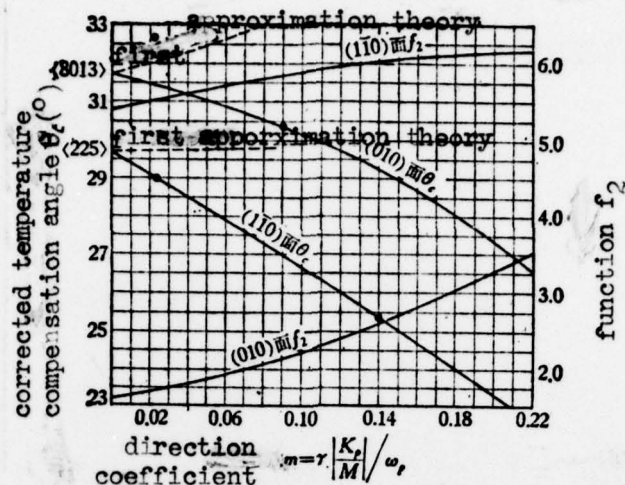


Figure 3 The relation of corrected anisotropy field temperature compensation direction in crystal plane of  $\{010\}$  kind and  $\{110\}$  kind and the direction coefficient  $m$  ( $m = \gamma \frac{K_2}{M} / \omega$ , and changes following the changes of frequency and temperature). There is also  $f_2$  value calculated according to equation (15) after the selection of  $\theta_c$  value for responding to different  $m$  value. The experimented points are mostly determined on the basis of Tokheim's data<sup>(4)</sup>.

But the result calculated by first approximation theory is contrary to this, and it predicts that the compensation angle be amplified following the amplification of  $m$ , and there is little change of the corrected compensation angle in crystal plane of  $\{1\bar{1}0\}$  kind. All these are not in accord with experiments. The experiment points in this Figure are mostly determined on the basis of Tokheim's data<sup>(4)</sup>. They did not clearly point out the problems of compensation direction correction but they decided a direction which has better temperature stability through experiment and by using a direction filter. In crystal plane of  $\{110\}$  kind, a better temperature compensation axis for a filter of 3-11 GHz of pure YIG sphere is  $\theta_c = 29^\circ$ ; for a filter of 1-11GHz of YIG sphere mixed with Ga, the better compensation direction



is  $\theta_c = 25.2^\circ$ . In the vicinity of plane  $\{010\}$ , the better compensation direction tested by using a filter of 2-10 GHz of Ga-YIG sphere is at  $\varphi = 3.8^\circ, \theta_c = 30.3^\circ$ ; if using a filter of 3-11 GHz of pure YIG sphere, the compensation direction will be at  $\varphi = 1.4^\circ, \theta_c = 32.7^\circ$ . These results basically are much the same as the theoretical analyses made here. Following the expansion toward low frequency of the filter and the increase of anisotropy field of the materials, the included angle between better compensation direction and axis  $\langle 001 \rangle$  becomes smaller; but the change of position angle in plane of  $\{110\}$  kind is great, so it is not good for temperature compensation. The only exception is the last result in the list, and compared with the third example, the possibility of error seems very great. This is because under higher frequency, when pure YIG material is used, the the temperature drift of the frequency is trivial, so it is easy to be affected by other factors in deciding compensation direction.

In practical directing, in order to make the instability of the devices within the range of whole frequency and temperature fall in a permitted limit, we can select a compromised  $m$  value for the single crystal YIG sphere and the frequency is selected at lower medium. For instance, in a  $\omega_r \leq \sqrt{\omega_1 \omega_2}$ ,  $\omega_1$  and  $\omega_2$  are the upper and lower operating bands of the device, and  $K_1/M$  of the material is to select a quantitative value leaning to the low temperature side within the range of working temperature.

#### 6. The Temperature Characteristics of YIG Devices When Working at Corrected Compensation Direction

As  $m$  is the function of frequency and temperature, it does not exist at the direction where the compensation can reach to an extent that the temperature coefficient is zero. From equation (20), the part where the



frequency follows temperature to change is

$$\frac{\delta\omega}{\gamma} = \frac{1}{2} f_1 \frac{K_1}{M} + \left( \frac{f_1}{2} - \frac{f_1^2}{8} \right) \frac{K_1^2}{M^2 H}$$

At the corrected temperature compensation direction,  $f_1$  and  $f_2$  can satisfy equation (26), and the part of  $f_1/8 \ll f_2/2$ , will change into

$$\delta\omega = \frac{\gamma^2 f_1}{2} \left| \frac{K_1}{M} \right| \left[ \frac{\left| \frac{K_1}{M} \right|}{\omega} - \frac{\left| \frac{K_2}{M} \right|}{\omega_p} \right]. \quad (28)$$

In Document (4) a similar formula was suggested, but analytic form of  $f_2$  was given. So it is rather difficult to calculate temperature relation unless some adjustable parameter is introduced. Here  $\delta\omega$  is the total skew difference of frequency which has skewed away from  $\omega = \gamma H$ , and it includes non-linear frequency skewness caused by anisotropy field and frequency transformation caused by change of temperature. If temperature remains unchanged, such as room temperature, the non-linear frequency skew value under different frequency will be

$$\delta\omega_k = \frac{\gamma^2 f_1}{2} \left| \frac{K_k}{M} \right| \left[ \frac{\left| \frac{K_k}{M} \right|}{\omega} - \frac{\left| \frac{K_p}{M} \right|}{\omega_p} \right]. \quad (29)$$

In it  $K_k/M$  indicates  $K_1/M$  value at room temperature. In practical test of different temperature, it often takes resonance frequency at room temperature as criterion in observing temperature drift of frequency. When temperature has changed, the resonance frequency which corresponds to the change of room temperature value is

$$\Delta\omega_T = \frac{\gamma^2 f_1}{2} \left| \frac{K_1}{M} \right| \left[ \frac{\left| \frac{K_1}{M} \right|}{\omega} - \frac{\left| \frac{K_p}{M} \right|}{\omega_p} \right] - \delta\omega_k, \quad (30)$$

This is corresponding to the results obtained from real observation.

In YIG device design, after tuned frequency range and YIG material have been selected, a pre-estimation of the temperature characteristics is very important. Due to the instability of the remanent effect of the magnetocrystalline anisotropy field, it can be calculated in the following ways: first selecting proper value of  $w_p$  and  $K_p/M$ , then calculate direction parameter  $m$ . From Figure 3 we can have corrected compensation direction  $\theta_c$ , then substitute  $\theta_c$  in equation (15) or (17) and (18) and calculate the value of  $f_2$  (in Figure 3, there is  $f_2$  value which can be used as direct reference). As for temperature, it usually takes room temperature to calculate  $\delta\omega_k$ . Using  $K_1/M$  value of materials under different temperature, based on equation (30), the temperature characteristics of the devices can be obtained through calculation. For example, to a device of 1-9 GHz (using Ga-YIG,  $4\pi M = 360$  gauss), if the additional magnetic field is in the corrected compensation direction in crystal plane of  $\{100\}$  kind and  $\{110\}$  kind ( $\theta_c = 30^\circ 22'$  and  $25^\circ 20'$ , they are respectively close to 10, 0, 17 and  $\langle 113 \rangle$  direction), the temperature characteristics can be seen in Figure 4. Because the low temperature anisotropy field of Ga-YIG is rather large, the temperature frequency change at 1 GHz is also large and it has gone beyond the practical range. Those theory curves, the experiments in Document (4) and the results of analysis are in good accord with each other. From Figure 4, it can be seen that the temperature stability of compensation direction in crystal plane of  $\{100\}$  kind is much better than that in crystal plane of  $\{110\}$  kind. This means that the best compensation direction mentioned above is the

compensation direction in crystal plane of  $\{100\}$  kind. After correction, all the corrected directions are still the best ones.

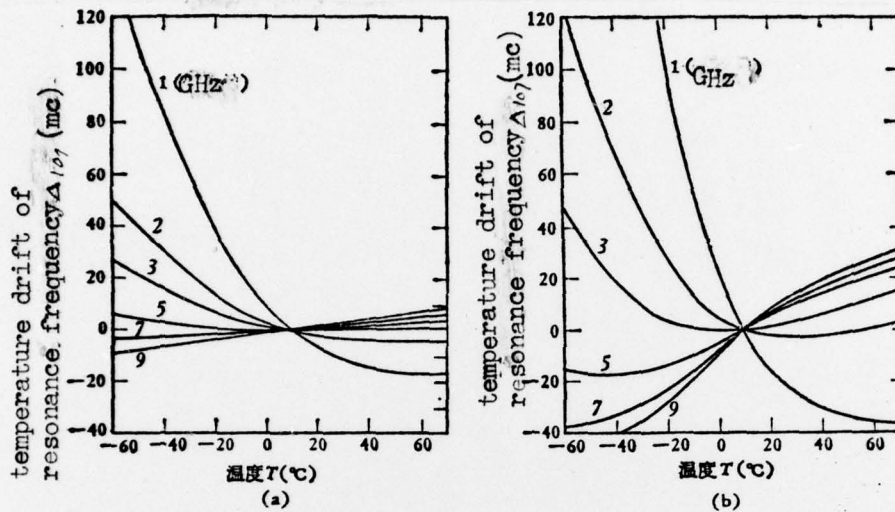


Figure 4 Temperature characteristics when YIG devices working at corrected temperature compensation direction

(a) Magnetic field is at compensation direction in crystal plane  $010$ ,  $\theta_c = 30^\circ 22'$ , close to  $10, 0, 17$  direction.  $m = 0.093$ ,  $\left| \frac{K_2}{M} \right| = 100$  Oe,  $w_p = 2000$  mc.

(b) Magnetic field is at compensation direction in crystal plane  $\{110\}$ ,  $\theta_c = 25^\circ 20'$ , close to direction  $\langle 113 \rangle$ .  $m = 0.14$ ,  $\left| \frac{K_2}{M} \right| = 100$  Oe,  $w_p = 2000$  mc. In tuned range of 1-9 GHz, using Ga-YIG material,  $4\pi M = 360$  gauss. Anisotropy field  $\left| \frac{K_1}{M} \right|$  (奥) = 125 (在  $-40^\circ\text{C}$ ), 100 ( $-20^\circ\text{C}$ ), 75 ( $0^\circ\text{C}$ ), 65 ( $10^\circ\text{C}$ ), 55 ( $20^\circ\text{C}$ ), 43 ( $40^\circ\text{C}$ ), 34 ( $60^\circ\text{C}$ )<sup>[4]</sup>

Using pure YIG single crystal at devices of high microwave frequency (larger than 4 GHz), the direction parameter  $m$  is often smaller than 0.05. Then the angular corrected quantity of compensation direction in crystal plane  $\{100\}$  is approximately equal to direction error. At this time, to make any correction at compensation direction obviously has little significance,



but it is permissible to take direction  $\langle 8013 \rangle$ . To Ga-YIG single crystal at devices of which the low frequency band is lower than 3 GHz, it is always necessary to make correction to the compensation direction. For instance, according to theoretical calculation, Ga-YIG filter of 3-9GHz, corrected the temperature compensation axis in crystal plane  $\{100\}$  working in a range of  $20-60^\circ$ , the largest temperature drift of the frequency is 6 mc, but working at direction  $\langle 8013 \rangle$  without correction, the temperature drift is 12 mc.

#### 7. The Difficulties in Low Microwave Frequency

In YIG devices of low microwave frequency, the anisotropy field of the material is large. For instance, YIG mixed with Ga, of which the saturated magnetization is 360 gauss, at  $-20^\circ$ ,  $\left| \frac{K_1}{M} \right| = 100$  Oe. At 1000 mc,  $m = 0.28$ , there practically is no more significant compensation direction, but there is serious non-linear effect. This brings great difficulty to low frequency band, and it can be seen in Figure 4. One way to overcome this difficulty is to place the YIG sphere constantly at higher temperature. Of course, the best way is to try to make new material, which has low saturated magnetization and low resonance linear width as required by low frequency, and magnetocrystalline anisotropy field which is close to zero. At the present time, there has been a possibility of requiring such material in some garnet ferrite, which is mixed with In, Zr and Sn. Because the anisotropy field is close to zero, the linear width of these multi-crystal materials is of only a few Oe<sup>(10,11)</sup>. Now the task in front of us is to make these new materials with single crystal of good quality and certainly we must anticipate some difficulty.

## Bibliography

- [1] J. Clark *et al.*, *IEEE Trans. on M. T. T.*, MTT-11 (1963), 447.
- [2] E. Czerlinsky *et al.*, *J. A. P.*, 36 (6) (1965), 1799.
- [3] R. E. Tokheim *et al.*, *IEEE Trans. on Mag.*, MAG-6 (1970), 583.
- [4] R. E. Tokheim, G. F. Johnson, *IEEE Trans. on Mag.*, MAG-7 (1971), 267.
- [5] C. Kittel, *Phys. Rev.*, 73 (1948), 155; *ibid.*, 76 (1949), 743.
- [6] L. R. Bickford, *Phys. Rev.*, 78 (1950), 449.
- [7] J. O. Artman, *Phys. Rev.*, 105 (1957), 62.
- [8] H. Suhl, *Phys. Rev.*, 97 (1955), 555.
- [9] C. R. Buffler, *Electronics Letters*, 2 (3) (1966), 116.
- [10] H. J. Van Hook *et al.*, *J. A. P.*, 40 (1969), 4001.
- [11] G. Winkler *et al.*, *Philips Res. Repts.*, 27 (1972), 151.

# DISTRIBUTION LIST

## DISTRIBUTION DIRECT TO RECIPIENT

<u>ORGANIZATION</u>	<u>MICROFICHE</u>	<u>ORGANIZATION</u>	<u>MICROFICHE</u>
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
B344 DIA/RDS-3C	9	E403 AFSC/INA	1
C043 USAMIIA	1	E404 AEDC	1
C509 BALLISTIC RES LABS	1	E408 AFWL	1
C510 AIR MOBILITY R&D	1	E410 ADTC	1
LAB/F10		E413 ESD	2
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NICD	3
C619 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NICD	2
H300 USAICE (USAREUR)	1		
P005 ERDA	1		
P005 CIA/CRS/ADB/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		